FINITE ELEMENT METHODS

The finite element method constitutes a general tool for the numerical solution of partial differential equations in engineering and applied science. advances the method with the development of digital computers. However, interest in approximate solutions of field equations of the classical field theories (e.g. elasticity,electro-magnetism) themselves on variational methods and the weighted-residual approach taken form the theoretical framework to the finite element method. With a bit of a stretch,one may even claim that Schellbach's approximate solution to Plateau's problem (find a surface of minimum area enclosed by a given closed boundary)

Mr. Clough therefore arrived a conclusion in his invention the finite element analysis particularly in structural elements of continuum it is divided into a finite number of elements having finite dimensions and reducing the continuum having infinite degree of freedom to finite degrees of freedom for analysis of structures

Advantages and disadvantages of Finite Element Method

ADVANTAGE

Mainly the physical problems intractable and complex for any closed bound can be analyzed by this method.

1. Efficiently applied to irregular geometry to get solution.
2. Any type of boundary objects can be analyzed
3. Anisotropy and In homogeneity of material can also be easily considered in this method.
4. Any type of loading can be considered and can be solved.

DISADVANTAGE

1. There are many type of problems where some other method can solove efficient than the finite element method
2. The cost involved in this method is very high.
3. In many cases of Vibration and stability the cost of analysis by this method should be avoided .
4. The approximation used in development of element stiffness naturally in this method.No element will represent all possible behavior pattern equally So compensation in one element causes distribution of another behavior.
5. The stress values may vary by 25% from fine mesh analysis to average mesh analysis.
6. The aspect ratio may affect the final results.(longer to smaller dimension of elements)
7. Interpretation of output problem arises in this method. So much of axial force and moment etc. are to be analyzed to produce a stack of output in some cases in frame analysis. Even a smaller problem in this method would generate so many numbers that has to be considered. Graphical representation of displacements and stresses to specific application require expertise.

In a Nut shell

There are four principles areas directly connected with the finite element method

1. Derivation of the Theory
2. Idealizing actual problem to approximate finite element problem
3. Computer program development for applying finite element theory
4. Investigation of date information processing and numerical methods needed to compute finite element solution

The major assumption on which the development is based should be understood and known to the engineers applying finite element method. Significance of approximation assumed and built in the method and the limitations of the analytical models developed should be known. This knowledge in finite element method reveals the difference between the good and bad disaster.:"

FINITE ELEMENT METHODS PRINCIPLES...

Finite Element Method is numerical method employed for obtaining solutions to many problems

Encountered in Engineering and Mathematical Physics field.

Engineering fields

1 Structural analysis

2 heat transfer

3 Fluid flows

4 Mass transfers

5 Electromagnetic potential

Application of this FEM analysis in

1. Design of Automobiles

2. Air frames

3. High rise building

4. Space crafts

5. Heat engines

6. Electric motors

7. Bearing ….etc.

Analytical solution is a mathematical expression that gives the values of desired unknown quantity at any location in a body. (Body i.e. Whole structure or any physical (area) system of interest.)

Consequently it is valid for infinite number of locations in that body.

Analytical method is employed for solving simple idealized problems.

Due to the

1 Complicated Geometry

2 Complicated Loadings

3 Complicated Material properties

It is very difficult or not at all possible to get solutions or cent percent correct solutions are not possible some times in engineering problems practically by analytical methods.

To overcome this method or alternative technique called NUMERICAL Methods used to find

At least Approximate Solutions. But it is accepted solution for complicated problems arise in engineering structural analysis.

Discretization or Finite element zing and their type

1. Variational method or Rayleigh –Ritz method

2. Weighted residual methods

3. Finite element method

4. Finite difference method

5. Finite volume method

6. Power series

7. Spectral methods and etc…

The variation method and weighted residual methods generally employ differential equation for solving the problems.

In the field of FEM technique *computers* are used to speed up the solution and solve very complex problem in a very short time.

FEM

A complex problem (region of continuing or Domain) - is discretized in to simple problems (i.e. Into simple geometric shapes or subdomains). These spilited domains are interconnected at some critical points.

Sub domains = finite elements

Inter connected points = nodal points or simply nodes

1. The material properties and 2. Governing relationships' (applied force and resultant displacement etc.)

These two things are imposed on the sub domains (finite elements) and suitable simultaneous equationsare formed for all elements.

The solutions of these equations give the approximate behaviors of the continuum (domain)

The sum of the elemental solutions will provide the required approximate solution for the whole domain or system.

FEM / FEA processes involve three stages of activity 1. PREPROCESSING 2. PROCESSING 3. POST PROCESSING.

PREPROCESSING - PREPRATION OF DATA (nodal co-ordinates, connectivity, boundary conditions, loading and material information)

PROCESSING – STIFFNESS GENERATION, STIFFNESS MODIFICATION AND SOLUTION OF EQUATION, RESULTING IN THE EVALUATION OF NODAL VARIABLES.

DERIVED QUANTITIES LIKE GRADIENTS OR STRESSES EVALUATED.

POST PROCESSING - PRESENTATION OF RESULTS.

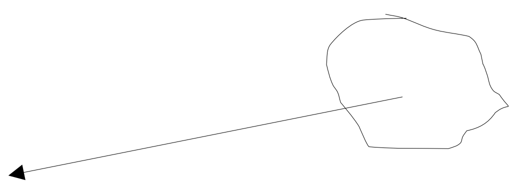
DEFORMED CONFIGUATION, STRESS DISTRIBUTIONS, TEMPERATURES ETC. ARE COMPUTED AND DISPLAYED.

concept of discretization

The basic goal of discretization is to provide an approximation of an infinite dimensional system by a system that can be fully defined with a finite number of "degrees of freedom".

To clarify the notion of dimensionality, consider a deformable body in the three-dimensional Euclidean space, for which the position of a typical particle with reference to a fixed coordinate system is defined by means of a vector x, as in Figure. This is an infinite dimensional system with respect to the position of all of its particle points. If the same body is assumed to be rigid, then it is a finite dimensional system with only six degrees of freedom.

A dimensional reduction of the above system is accomplished by placing a (somewhat severe)



Structural analogue substitution method

Consider the oscillation of a liquid in a manometer. This system can be approximated ("lumped") by means of a single degree-of-freedom mass-spring system, as in Figure

Clearly, such an approximation is largely intuitive and cannot precisely capture the complexity of the original system (viscosity of the liquid, surface tension effects, geometry of the manometer walls).

Structural analogue substitution method

Consider the oscillation of a liquid in a manometer. This system can be approximated ("lumped") by means of a single degree-of-freedom mass-spring system, as in Figure

Clearly, such an approximation is largely intuitive and cannot precisely capture the complexity

of the original system (viscosity of the liquid, surface tension effects, geometry of the manometer walls). example of the structural analogue method

BASIC STEPS OF FEM

1 Discretization of the structure

2 Selection of Displacement function

3 Formation of the element stiffness matrix and load vector

4 Formation of Global stiffness matrix and load vector

5 Incorporation of Boundary conditions

6 Solution of Simultaneous equations

7 Calculation of element strains and stresses

8 Interpretation of the result obtained.

Discretization or Finite element zing and their type

1. Based on Dimension

2. Based on Material Property

3. Based on Degree of Freedom

1 Based on Dimension

One dimensional element - line element , two dimensional element – Triangular and quadrilateral, three dimensional elements - Tetrahedral and hexahedral elements

2 Based on Material Property

Linear element , non linear element

3 Based on Degree of Freedom

Translational - one or two or three degree of freedom

Rotational - one or two or three degree of freedom.

Category of finite elements

Simple , complex and multiple elements

The order of polynomial used in interpolation function decided by the geometry of the element

For

Simple elements - Interpolation function contains only constant and linear terms only.

Ie. **Approximating polynomial expression has only constant and linear terms in simplex element. So they are called linear elements.**

The elements specified for n=1 single order polynomial for one two and three dimensional elements.

The simplex element of one dimensional is a line with two nodes at the ends

Material property

Material Behaviour

Material of structure and machine parts undergoing deformation due to external load that causes stress due to force and displacement due to strain in the body or structure according to Hooks law of stress and strain

Stress α Strain   with the condition of material type 1 Non linear 2 linear 3 In   elastics

Boundary condition

Structure cannot resist external load or its own self-weight without any proper boundary conditions. So all the structural problems are analyzed with in the value of min and max and solved by Boundary value problems with analytical method of integration .

Degree of freedom

According to the type of problem structures are categorized as 1 Having discrete element only 2. Those which are continuium 3. Those which have both discrete element and continuium . By this classification some amount of ease is brought in degree of freedom or to have known deformation and forces in discrete element. In continuum degree of freedom is applied with FE since it is difficult to treat infinite degree of freedom as it have infinite as max value.

Concept

Continuous fields are represented by piecewise linear, quadratic or cubic field over the set of discretized subdomains. Pictorial idea of this cantilever beam is considered.

concept of discretization

The basic goal of discretization is to provide an approximation of an infinite dimensional

system by a system that can be fully defined with a finite number of “degrees of freedom”.

To clarify the notion of dimensionality, consider a deformable body in the three-dimensional

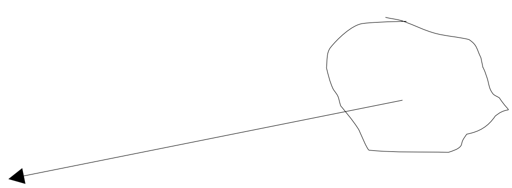
Euclidean space, for which the position of a typical particle with reference to a fixed coordinate

system is defined by means of a vector x, as in Figure. This is an infinite

dimensional system with respect to the position of all of its particle points. If the same body

is assumed to be rigid, then it is a finite dimensional system with only six degrees of freedom.

A dimensional reduction of the above system is accomplished by placing a (somewhat severe)



Structural analogue substitution method

Consider the oscillation of a liquid in a manometer. This system can be approximated

(“lumped”) by means of a single degree-of-freedom mass-spring system, as in Figure 1.2.

Clearly, such an approximation is largely intuitive and cannot precisely capture the complexity

of the original system (viscosity of the liquid, surface tension effects, geometry of the

manometer walls).

example of the structural analogue method

==

example of the structural analogue method

The structural analogue substitution method, whenever applicable, generally provides coarse approximations to complex systems. However, its degree of sophistication (hence, also the fidelity of its results) can vary widely. The “network analysis” of Kron is generally viewed as a typical example of the structural analogue approach.

Kd 2u dx2 = f in (0, L) , (1.1)

u(0) = u0 , (1.2)

u(L) = uL , (1.3)

where k is a constant and f = f(x) is a smooth function. Assume that N points are chosen

in the interior of the domain (0, L), each of them equidistant from its immediate neighbors.

An algebraic (or “difference”) approximation to the second derivative may be computed as

d 2u dx2

Finite difference method

Consider the ordinary differential equation

Kd 2u dx2 = f in (0, L) , (1.1)

u(0) = u0 , (1.2)

u(L) = uL , (1.3)

where k is a constant and f = f(x) is a smooth function. Assume that N points are chosen

in the interior of the domain (0, L), each of them equidistant from its immediate neighbors.

An algebraic (or “difference”) approximation to the second derivative may be computed as

d 2u dx2l ≈ ul+1 − 2ul + ul−1 Δx 2 , (1.4) with error o(Δx 2). Indeed, employing twice a Taylor series expansion with remainder around

0 1 l−1 l l+1 N N+1 ≈ ul+1 − 2ul + ul−1 Δx 2 , (1.4) with error o(Δx 2). Indeed, employing twice a Taylor series expansion with remainder around

restriction on the admissible motions that the body may undergo.

Finite dimensional approximations are very important from the computational standpoint,

because they often allow for analytical and/or numerical solutions to problems that would otherwise be intractable. There exist various methods that can reduce infinite dimensional systems to approximate finite dimensional counterparts. Here we consider three of

those methods, namely the physically motivated structural analogue substitution method, the finite difference method and finite element method.

==================================

The structural analogue substitution method, whenever applicable, generally provides coarse approximations to complex systems. However, its degree of sophistication (hence, also the fidelity of its results) can vary widely. The “network analysis” of Kron is generally viewed as a typical example of the structural analogue approach.

Kd 2u dx2 = f in (0, L) , (1.1)

u(0) = u0 , (1.2)

u(L) = uL , (1.3)

where k is a constant and f = f(x) is a smooth function. Assume that N points are chosen

in the interior of the domain (0, L), each of them equidistant from its immediate neighbors.

An algebraic (or “difference”) approximation to the second derivative may be computed as

d 2u dx2

Finite difference method

Consider the ordinary differential equation

Kd 2u dx2 = f in (0, L) , (1.1)

u(0) = u0 , (1.2)

u(L) = uL , (1.3)

where k is a constant and f = f(x) is a smooth function. Assume that N points are chosen

in the interior of the domain (0, L), each of them equidistant from its immediate neighbors.

An algebraic (or “difference”) approximation to the second derivative may be computed as

d 2u dx2l ≈ ul+1 − 2ul + ul−1 ∆x 2 , (1.4) with error o(∆x 2). Indeed, employing twice a Taylor series expansion with remainder around

0 1 l−1 l l+1 N N+1 ≈ ul+1 − 2ul + ul−1 Δx 2 , (1.4) with error o(Δx 2). Indeed, employing twice a Taylor series expansion with remainder around

restriction on the admissible motions that the body may undergo.

Finite dimensional approximations are very important from the computational standpoint,

because they often allow for analytical and/or numerical solutions to problems that

would otherwise be intractable. There exist various methods that can reduce infinite dimensional

systems to approximate finite dimensional counterparts. Here we consider three of

those methods, namely the physically motivated structural analogue substitution method, the finite

difference method and finite element method.

The finite element method provides a general procedure for the construction of admissible

spaces and in connection with the weighted-residual and variational methods discussed .

With reference to the general form of the approximation functions one may establish a distinction

between global and local approximation methods. Local approximation methods are those for which

supp ϕI is "small" compared to the size of the domain of approximation, whereas global methods

employ interpolation functions with relatively "large" support.

Global and local approximation methods present both advantages and disadvantages.

Global methods are often capable of providing excellent estimates of a solution with relatively

small computational effort, especially when the analyst has a good understanding of the

expected solution characteristics. However, a proper choice of global interpolation functions

may not always be readily available, as in the case of complicated domains, where satisfaction

of any boundary conditions could be a difficult, if not an insurmountable task.

In addition, 67 DRAFT and 68 Finite element subspaces

global methods rarely lend themselves to a straightforward algorithmic implementation, and

even when they do, they almost invariably yield dense linear systems which may require substantial

computational effort to solve. Local methods are more suitable for algorithmic implementation than

global methods, as they can easily satisfy Dirichlet (or essential) boundary conditions, and they typically yield “banded” linear algebraic systems. Moreover, these methods are flexible in allowing local refinements in the approximation, when warranted by the analysis. However, local methods can be surprisingly expensive, even for simple problems, when the desired degree of accuracy is high. The so-called global-local approximation methods combine both global and local interpolation functions in order to exploit the positive characteristics of both methods.

Interpolation functions that appear in equations (3.10) and (3.11) need to satisfy certain general admissibility criteria. These criteria are motivated by the requirement that the resulting finite-dimensional solution spaces be well-defined and capable of accurately and uniformly approximating the exact solutions. In particular, all families of interpolation functions {ϕ1, . . . , ϕN } should have the following properties:

(a) For any x ∈ Ω, there exists an I with 1 ≤ I ≤ N, such that ϕI (x) 6= 0. In other words, the interpolation functions should “cover” the whole domain of analysis. Indeed, if the above property is not satisfied, it follows that there exist interior points of Ω where the exact solution cannot be approximated.

(b) All interpolation functions should satisfy the Dirichlet (or essential) boundary conditions, if required by the underlying weak form, as discussed in Chapters 3 and 4.

(c) The interpolation functions should be linearly independent in the domain of analysis.

To further elaborate on this point, let Uh be the space of admissible solutions spanned by functions {ϕ1, . . . , ϕN }, namely

Uh = {uh | uh = X N I=1

αI ϕI , αI ∈ R, I = 1, . . . , N} .

Linear independence of the interpolation functions is equivalent to stating that given any uh ∈ Uh, there exists a unique set of parameters {α1, . . . , αN }, such that

uh = X N I=1 αI ϕI .

αI ϕI = 0 ⇔ αI = 0 , I = 1, . . . , N .

If property (c) holds, then functions {ϕ1, . . . , ϕN } are said to form a basis of Uh.

Linear independence of the interpolation functions is essential for the derivation of approximate solutions. Indeed, if parameters $\{\alpha_1, \ldots, \alpha_N\}$ are not uniquely defined for any given $u_h \in U_h$, then the linear algebraic system does not possess a unique solution and, consequently, the discrete problem is ill-posed.

(d) Interpolation functions must satisfy the integrability requirements emanating from the associated weak forms, as discussed.

(e) The family of interpolation functions should possess sufficient "approximating power".

One of the most important features of Hilbert spaces is that they provide a suitable framework for examining the issue of how (and in what sense) a function $u_h \in U_h \subset U$, defined as u_h = X NI=1 $\alpha_I \phi_I$

approximates a function $u \in U$ as N increases. In order to address the above point, consider a set of functions $\{\phi_1, \phi_2, \ldots, \phi_N, \ldots\}$, which are linearly independent in U and, thus, form a countably infinite basis.[1] These functions are termed orthonormal in U if $< \phi_I, \phi_J >$ = 1 if I = J 0 if I 6= J.

Any countably infinite basis can be orthonormalized by means of a Gram-Schmidt orthogonalization procedure, as follows: starting with the first function ϕ_1, let

$\psi_1 := \phi_1\ k\phi_1 k$,

so that, clearly,

$< \psi_1, \psi_1 > = 1$.

Then, let

$\psi_2 = a_2[\phi_2 - < \phi_2, \psi_1 > \psi_1]$, (5.1)

[1]Hilbert spaces can be shown to always possess such a basis.

where a_2 is a scalar parameter to be determined. It is immediately seen from (5.1) that

$< \psi_1, \psi_2 > = < \psi_1, a_2\phi_2 - a_2 < \phi_2, \psi_1 > \psi_1 >$

$= a_2 < \psi_1, \phi_2 > - a_2 < \psi_1, \psi_1 >< \psi_1, \phi_2 > = 0$.

The scalar parameter a_2 is determined so that $k\psi_2 k = 1$, namely

$a_2 = 1 \, k\phi_2 - < \phi_2, \psi_1 > \psi_1 k$.

In general, the function ϕ_{K+1}, $K = 1, 2, \dots,$ gives rise to ψ_{K+1} defined as

$\psi_{K+1} = a_{K+1}[\phi_{K+1} - X \, KI=1 < \phi_{K+1}, \psi_I > \psi_I \,]$

where $a_{K+1} = 1k\phi_{K+1} - XKI=1 < \phi_{K+1}, \psi_I > \psi_I k$.

To establish that $\{\psi_1, \psi_2, \dots, \psi_N, \dots\}$ are orthonormal, it suffices to show by induction

that if $\{\psi_1, \psi_2, \dots, \psi_K\}$ are orthonormal, then ψ_{K+1} is orthonormal with respect to

each of the first K members of the sequence. Indeed, using (5.2) it is seen that

$< \psi_{K+1}, \psi_K > = < a_{K+1}\phi_{K+1} - a_{K+1} \, X \, K \, I = 1 < \phi_{K+1}, \psi_I > \psi_I \, , \psi_K >$

$= < a_{K+1}\phi_{K+1}, \psi_K > - KX-1I = 1 \, a_{K+1} < \phi_{K+1}, \psi_I >< \psi_I \, , \psi_K > - a_{K+1} < \phi_{K+1}, \psi_K >< \psi_K, \psi_K > = 0$

and, for $N < K$,

$< \psi_{K+1}, \psi_N > = < a_{K+1}\phi_{K+1} - a_{K+1} \, X \, K \, I = 1 < \phi_{K+1}, \psi_I > \psi_I \, , \psi_N >$

$= < a_{K+1}\phi_{K+1}, \psi_N > - N \, X-1I=1 \, a_{K+1} < \phi_{K+1}, \psi_I >< \psi_I \, , \psi_N > - XKI$

$=N+1a_{K+1} < \phi_{K+1}, \psi_I >< \psi_I \, , \psi_N > - a_{K+1} < \phi_{K+1}, \psi_N >< \psi_N \, , \psi_N > = 0$, .